

# Probabilistic Wind Speed Forecasting using Ensembles and Bayesian Model Averaging

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## Abstract

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# 1 Introduction

While deterministic point forecasts have long been the standard in weather forecasting, there are many situations in which probabilistic information can be of value. In this paper, we consider the case of wind speed. Often, ranges or thresholds can be of interest — sailors are likely to be more interested in the probability of there being enough wind to go out sailing than in simply the best guess at the wind speed, and farmers may be interested in the chance of winds being low enough to safely spray pesticides. Possible extreme values are of particular interest, where it can be important to know the chance of winds high enough to pose dangers for boats or aircraft.

In situations calling for a cost/loss analysis, the probabilities of different outcomes need to be known. For wind speed, this issue often arises in the context of wind power, where underforecasting and overforecasting carry different financial penalties. The optimal point forecast in these situations is often a quantile of the predictive distribution (Roulston et al. 2003; Pinson et al. 2007; Gneiting 2008). Different situations can require different quantiles, and this needed flexibility can be provided by forecasts of a full predictive probability density function (PDF). Environmental concerns and climate change have made wind power look like an appealing source of clean and renewable energy, and as this field continues to grow, calibrated and sharp probabilistic forecasts can help to make wind power a more financially competitive alternative.

Purely statistical methods have been applied to short-range forecasts for wind speeds only a few hours into the future (Brown et al. 1984; Kretzschmar et al. 2004; Gneiting et al. 2006; Genton and Hering 2007). A detailed survey of the literature on short-range wind forecasting can be found in Giebel et al. (2003).

Medium-range forecasts looking several days ahead are generally based on numerical weather prediction models, which can then be statistically postprocessed. To estimate the predictive distribution of a weather quantity, an ensemble forecast is often used. An ensemble forecast consists of a set of multiple forecasts of the same quantity, based on different estimates of the initial atmospheric conditions and/or different physical models (Palmer 2002; Gneiting and Raftery 2005). An example of an ensemble forecast for wind speed can be seen in Figure 1. Ensemble forecasts can give an indication of uncertainty, and a statistical relationship between forecast errors and ensemble spread has been established for several ensemble systems. However, it has also been shown that ensemble forecasts typically are uncalibrated, with a tendency for observed values to fall outside of the range of the ensemble too often (Grimm and Mass 2002; Buizza et al. 2005; Gneiting and Raftery 2005).

In this light, a number of methods have been proposed for statistically postprocessing

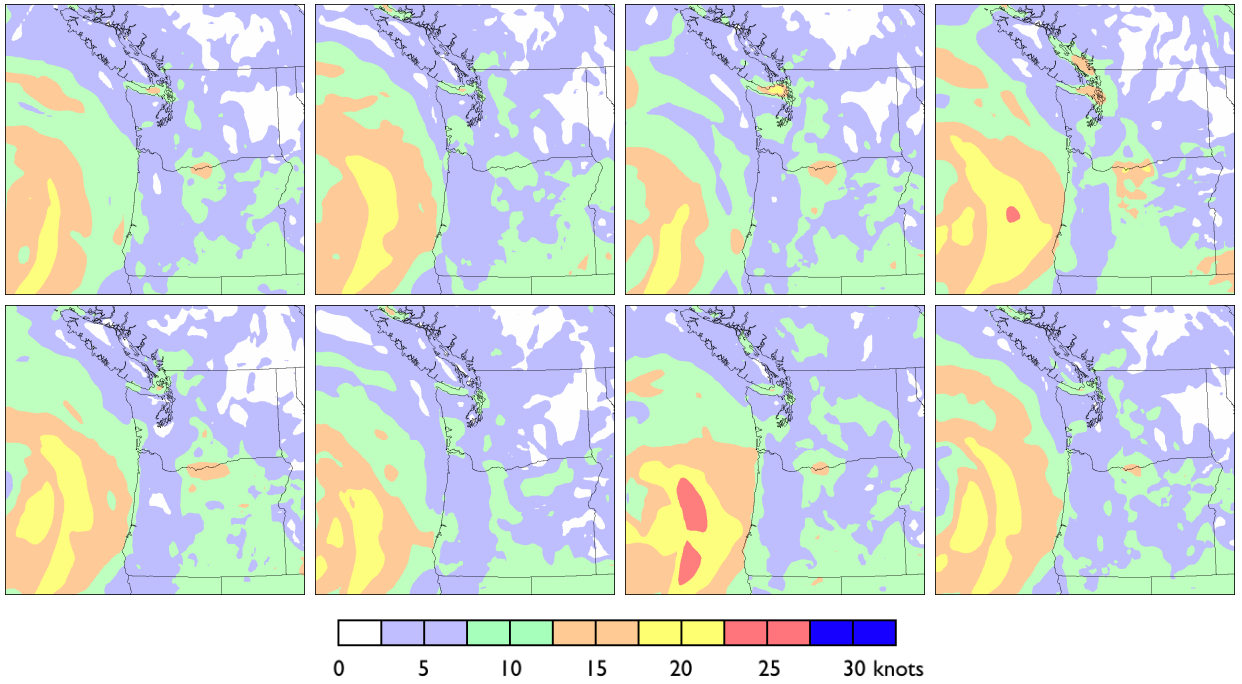


Figure 1: 48-hour ahead ensemble forecast of maximum wind speed over the Pacific Northwest on 7 August 2003 using the eight-member University of Washington mesoscale ensemble (Grimit and Mass 2002; Eckel and Mass 2005).

ensemble forecasts of wind speed or wind power. These approaches have largely focused on the use of quantile regression to generate forecast bounds and/or intervals (Bremnes 2004; Nielsen et al. 2006; Pinson et al. 2007; Møller et al. 2008). These methods do not, however, yield a full PDF; rather, they give only probabilities for certain specific events.

Bayesian model averaging (BMA) was introduced by Raftery et al. (2005) as a statistical postprocessing method for producing probabilistic forecasts from ensembles in the form of predictive PDFs. The BMA predictive PDF of any future weather quantity of interest is a weighted average of PDFs centered on the individual bias-corrected forecasts, where the weights can be interpreted as posterior probabilities of the models generating the forecasts and reflect the forecasts' contributions to overall forecasting skill over a training period. The original development of BMA by Raftery et al. (2005) was for weather quantities whose predictive PDFs are approximately normal, such as temperature and sea-level pressure. This method was modified by Sloughter et al. (2007) to apply to quantitative precipitation forecasts. These forecasts used component distributions that had a positive probability of being equal to zero, and, when not zero, were skewed, and modeled using power-transformed gamma distributions.

As with precipitation, wind speed has a skewed distribution. Unlike for precipitation,

there is no need to model the separate probability of wind speed being equal to zero, at least in the geographic region we consider. Here we develop a BMA method for wind speed by modeling the component distribution for a given ensemble member as a gamma distribution; the BMA PDF is then itself a mixture of such distributions.

In Section 2 we review the BMA technique and describe our extension of it to wind speed. Statistical approaches to wind forecasting offer a unique challenge in that observed values are reported discretized to the nearest whole knot, a much coarser discretization than is seen in other weather quantities. We compare a number of methods for estimating the parameters of the BMA PDF which account for the discretization in different ways. Then in Section 3 we give results for 48-hour ahead forecasts of maximum wind speed over the North American Pacific Northwest in 2003 based on the eight-member University of Washington mesoscale ensemble (Grimit and Mass 2002; Eckel and Mass 2005). Throughout the paper we use illustrative examples drawn from these data, and we find that BMA is calibrated and sharp for the period we consider. Finally, in Section 4 we discuss alternative approaches and possible improvements to the method.

## 2 Data and Methods

### 2.1 Forecast and observation data

This research considers 48-hour ahead forecasts of maximum wind speed over the Pacific Northwest in the period from 1 November 2002 through 31 December 2003, using the eight-member University of Washington mesoscale ensemble (Eckel and Mass 2005) initialized at 00 hours UTC, which is 5pm local time in summer, when daylight saving time operates, and 4pm local time otherwise. The dataset contains observations and forecasts at surface airway observation (SAO) stations, a network of automated weather stations located at airports throughout the United States and Canada. Maximum wind speed is defined as the maximum of the hourly ‘instantaneous’ wind speeds over the previous eighteen hours, where an hourly ‘instantaneous’ wind speed is a 2-minute average from the period of two minutes before the hour to on the hour. Data were available for 340 days, and data for 86 days during this period were unavailable. In all, 35,230 station observations were used, an average of about 104 per day. The forecasts were produced for observation locations by bilinear interpolation from forecasts generated on a twelve kilometer grid, as is common practice in the meteorological community. The wind speed observations were subject to the quality control procedures described by Baars (2005).

The wind speed data we analyze are discretized when recorded — wind speed is rounded to the nearest whole knot. Additionally, any wind speeds below one knot are recorded as

zero. One knot is equal to approximately 0.514 meters per second, or 1.151 miles per hour.

## 2.2 Bayesian model averaging

Bayesian model averaging (BMA) (Leamer 1978; Kass and Raftery 1995; Hoeting et al. 1999) was originally developed as a way to combine inferences and predictions from multiple statistical models, and was applied to statistical linear regression and related models in the social and health sciences. Raftery et al. (2005) extended BMA to ensembles of deterministic prediction models and showed how it can be used as a statistical postprocessing method for forecast ensembles, yielding calibrated and sharp predictive PDFs of future weather quantities.

In BMA for forecast ensembles, each ensemble member forecast  $f_k$  is associated with a component PDF,  $g_k(y|f_k)$ . The BMA predictive PDF for the future weather quantity,  $y$ , is then a mixture of the component PDFs, namely

$$p(y|f_1, \dots, f_K) = \sum_{k=1}^K w_k g_k(y|f_k), \quad (1)$$

where the BMA weight  $w_k$  is based on forecast  $k$ 's relative performance in the training period. The  $w_k$ 's are probabilities and so they are nonnegative and add up to 1, that is,  $\sum_{k=1}^K w_k = 1$ . Here  $K$  is the number of ensemble members.

The component PDF  $g_k(y|f_k)$  can be thought of roughly as the conditional PDF of the weather quantity  $y$  given the  $k$ th forecast,  $f_k$ , conditional on  $f_k$  being the best forecast in the ensemble. This heuristic interpretation is in line with how operational weather forecasters often work, by selecting one or a small number of 'best' forecasts from a potentially large number available, based on recent predictive performance (Joslyn and Jones 2008).

## 2.3 Gamma model

For weather variables such as temperature and sea level pressure, the component PDFs can be fit reasonably well using a normal distribution centered at a bias-corrected forecast, as shown by Raftery et al. (2005). For precipitation, Sloughter et al. (2007) modeled the component PDFs using a mixture of a point mass at zero and a power-transformed gamma distribution.

Haslett and Raftery (1989) modeled the square root of wind speeds using a normal distribution. Wind speed distributions have also often been modeled by Weibull densities (Justus et al. 1976; Hennessey 1977; Justus et al. 1978; Stevens and Smulders 1979). Tuller and Brett (1984) noted that the necessary assumptions for fitting a Weibull distribution are not always met. Here we generalize the Weibull approach by considering gamma distribution fits

to power transformations of the observed wind speeds. We found that gamma distributions for the raw observed wind speeds gave a good fit, and fit better than using any power transformation. In light of this, we model the component PDFs of wind speed as untransformed gamma distributions. The gamma distribution with shape parameter  $\alpha$  and scale parameter  $\beta$  has the PDF

$$g(y) = \frac{1}{\beta^\alpha \Gamma(\alpha)} y^{\alpha-1} \exp(-y/\beta) \quad (2)$$

for  $y > 0$ , and  $g(y) = 0$  for  $y \leq 0$ . The mean of this distribution is  $\mu = \alpha\beta$ , and its variance is  $\sigma^2 = \alpha\beta^2$ .

It remains to specify how the parameters of the gamma distribution depend on the numerical forecast. An exploratory data analysis showed that the observed wind speed is approximately linear as a function of the forecasted wind speed, with a standard deviation that is also approximately linear as a function of the forecast.

Putting these observations together, we get the following model for the component gamma PDF of wind speed:

$$g_k(y|f_k) = \frac{1}{\beta_k^{\alpha_k} \Gamma(\alpha_k)} y^{\alpha_k-1} \exp(-y/\beta_k). \quad (3)$$

The parameters of the gamma distribution depend on the ensemble member forecast,  $f_k$ , through the relationships

$$\mu_k = b_{0k} + b_{1k}f_k, \quad (4)$$

and

$$\sigma_k = c_{0k} + c_{1k}f_k, \quad (5)$$

where  $\mu_k = \alpha_k\beta_k$  is the mean of the distribution, and  $\sigma_k = \sqrt{\alpha_k}\beta_k$  is its standard deviation. Here we restrict the standard deviation parameters to be constant across all ensemble members. This simplifies the model by reducing the number of parameters to be estimated, makes parameter estimation computationally easier, and reduces the risk of overfitting, and we found that it led to no degradation in predictive performance. The  $c_{0k}$  and  $c_{1k}$  terms are replaced with  $c_0$  and  $c_1$ .

Our final BMA model for the predictive PDF of the weather quantity,  $y$ , here the maximum wind speed, is thus (1) with  $g_k$  as defined in (3).

## 2.4 Parameter estimation

Parameter estimation is based on forecast-observation pairs from a training period, which we take here to be the  $N$  most recent available days preceding initialization. The training period is a sliding window, and the parameters are reestimated for each new initialization period. We considered training periods ranging from the past 20 to 45 days. An examination

of the sensitivity of our results to training period length showed very similar performance across potential training period lengths. Differences in average errors were only seen three decimal places out. Within this range, we consistently saw that a 25 day training period gave very slightly better performance, and we will present results here based on this period.

### 2.4.1 Standard method

We first consider a standard method of parameter estimation similar to the method used for quantitative precipitation in Sloughter et al. (2007). We estimate the mean parameters,  $b_{0k}$  and  $b_{1k}$ , by linear regression. These parameters are member-specific, and are thus estimated separately for each ensemble member, using the observed wind speed as the dependent variable and the forecasted wind speed,  $f_k$ , as the independent variable.

We estimate the remaining parameters,  $w_1, \dots, w_K$ ,  $c_0$ , and  $c_1$ , by maximum likelihood from the training data. Assuming independence of forecast errors in space and time, the log-likelihood function for the BMA model is

$$\ell(w_1, \dots, w_K; c_0; c_1) = \sum_{s,t} \log p(y_{st} | f_{1st}, \dots, f_{Kst}), \quad (6)$$

where the sum extends over all station locations,  $s$ , and times,  $t$ , in the training data. As noted above, wind speed observations below one knot are recorded as zero knots. The log-likelihood requires calculating the logarithm of each observed wind speed, which is not possible with values of zero.

Wilks (1990) suggested a method for maximum likelihood estimation of gamma distribution parameters with data containing zeroes due to rounding by, for each value of zero, replacing the corresponding component of the log-likelihood with the aggregated probability of the range of values that would be rounded to zero (in our case, between 0 and 1 knots). To incorporate this, for each observed  $y_{st}$  recorded as a zero, we replace  $p(y_{st} | f_{1st}, \dots, f_{Kst})$  above with

$$p(y_{st} | f_{1st}, \dots, f_{Kst}) = P(1 | f_{1st}, \dots, f_{Kst}), \quad (7)$$

where

$$P(a | f_{1st}, \dots, f_{Kst}) = \int_0^a p(y | f_{1st}, \dots, f_{Kst}) dy. \quad (8)$$

The log-likelihood function cannot be maximized analytically, and instead we maximize it numerically using the EM algorithm (Dempster et al. 1977; McLachlan and Krishnan 1997). The EM algorithm is iterative, and alternates between two steps, the expectation (E) step, and the maximization (M) step. It uses the unobserved quantities  $z_{kst}$ , which are latent variables equal to 1 if observation  $y_{st}$  comes from the  $k$ th mixture component, and to 0 otherwise.

In the E step, the  $z_{kst}$  are estimated given the current estimate of the parameters. Specifically,

$$\hat{z}_{kst}^{(j+1)} = \frac{w_k^{(j)} p^{(j)}(y_{st}|f_{kst})}{\sum_{l=1}^K w_l^{(j)} p^{(j)}(y_{st}|f_{lst})}, \quad (9)$$

where the superscript  $j$  refers to the  $j$ th iteration of the EM algorithm, and thus  $w_k^{(j)}$  refers to the estimate of  $w_k$  at the  $j$ th iteration. The quantity  $p^{(j)}(y_{st}|f_{kst})$  is defined using the estimates of  $c_0$  and  $c_1$  from the  $j$ th iteration, and is either  $g_k(y_{st}|f_{kst})$  as defined in (3), if  $y_{st}$  is non-zero, or is  $G_k(1|f_{kst})$ , if  $y_{st}$  is zero, where

$$G_k(a|f_{kst}) = \int_0^a g_k(y|f_{kst}) dy. \quad (10)$$

Note that, although the  $z_{kst}$  are equal to either 0 or 1, the  $\hat{z}_{kst}$  are real numbers between 0 and 1. The  $\hat{z}_{1st}, \dots, \hat{z}_{Kst}$  are nonnegative and sum to 1 for each  $(s, t)$ .

The M step then consists of maximizing the expected log-likelihood as a function of  $w_1, \dots, w_K$ ,  $c_0$ , and  $c_1$ , where the expectation is taken over the distribution of  $z_{kst}$  given the data and the previous estimates. This is the same as maximizing the log-likelihood given the  $z_{kst}$  as well as  $w_1, \dots, w_K$ ,  $c_0$ , and  $c_1$ , evaluated at  $z_{kst} = \hat{z}_{kst}^{(j+1)}$ . Thus

$$w_k^{(j+1)} = \frac{1}{n} \sum_{s,t} \hat{z}_{kst}^{(j+1)}, \quad (11)$$

where  $n$  is the number of cases in the training set, that is, the number of distinct values of  $(s, t)$ . There are no analytic solutions for the M step estimates of the parameters  $c_0$  and  $c_1$ , and so they must be found numerically.

The E and M steps are then iterated to convergence, which we define as a change no greater than some small tolerance in the log-likelihood in one iteration. The log-likelihood is guaranteed to increase at each EM iteration (Wu 1983), which implies that in general it converges to a local maximum of the likelihood. Convergence to a global maximum cannot be guaranteed, so the solution reached by the algorithm can be sensitive to the starting values. Choosing the starting values based on equal weights and the marginal variance usually led to a good solution in our experience.

#### 2.4.2 Fully discretized method

While our standard method addresses the computational issues associated with recorded observations of zero, it does not more generally address the discretization of the wind speed values. We therefore consider a fully discretized method of parameter estimation that generalizes the method of Wilks (1990). Each component of the log-likelihood is replaced with the aggregated probability of the range of values that would be rounded to the recorded value.

In our case, a recorded observation of 0 indicates a true value between 0 and 1, a recorded observation of 1 indicates a true value between 1 and  $\frac{3}{2}$ , and for any integer  $i > 1$ , a recorded observation of  $i$  indicates a true value between  $i - \frac{1}{2}$  and  $i + \frac{1}{2}$ .

To extend the approach of our initial method, we first leave  $p(y_{st}|f_{1st}, \dots, f_{Kst})$  as defined in (7) for observed values of 0. For observed values of 1, we have

$$p(y_{st}|f_{1st}, \dots, f_{Kst}) = P(\frac{3}{2}|f_{1st}, \dots, f_{Kst}) - P(1|f_{1st}, \dots, f_{Kst}), \quad (12)$$

and for observed values  $i$  where  $i > 1$ ,

$$p(y_{st}|f_{1st}, \dots, f_{Kst}) = P(i + \frac{1}{2}|f_{1st}, \dots, f_{Kst}) - P(i - \frac{1}{2}|f_{1st}, \dots, f_{Kst}). \quad (13)$$

Analogously, in the E step of the EM algorithm, for observed values of 0, we put

$$p^{(j)}(y_{st}|f_{kst}) = G_k(1|f_{kst}), \quad (14)$$

for observed values of 1,

$$p^{(j)}(y_{st}|f_{kst}) = G_k(\frac{3}{2}|f_{kst}) - G_k(1|f_{kst}), \quad (15)$$

and for observed values  $i$  where  $i > 1$ ,

$$p^{(j)}(y_{st}|f_{kst}) = G_k(i + \frac{1}{2}|f_{kst}) - G_k(i - \frac{1}{2}|f_{kst}). \quad (16)$$

The rest of the EM algorithm remains unchanged.

### 2.4.3 Doubly discretized method

In the fully discretized method, the discretization of observations is taken into account in the log-likelihood. This allows us to account for the discretization when estimating the BMA weights and the standard deviation parameters, which are estimated via maximum likelihood. However, this does not address the mean parameters, which are fit via linear regression. To take account of the discretization in estimating the mean parameters, we additionally discretize the forecasts in the same manner that the observations have been. The parameters are then estimated as in the fully discretized method, replacing the ensemble member forecasts with the discretized forecasts.

### 2.4.4 Pure maximum likelihood method

We next investigate the possibility of estimating the mean parameters by maximum likelihood as well. To avoid computational problems with a parameter space of too high a dimension, we restrict the mean parameters to be constant across ensemble members, similarly to the

constraint already placed on the standard deviation parameters. We then estimate the BMA weights, mean parameters, and standard deviation parameters simultaneously via maximum likelihood, using the discretized log-likelihood function from the fully discretized method. The log-likelihood is optimized numerically.

#### **2.4.5 Parsimonious method**

We finally consider one additional method, taking the partially discretized log-likelihood from the standard method but adding the constraint that the mean parameters must be constant across ensemble members. This represents the most parsimonious model, in that it has the smallest number of parameters.

### **3 Results**

We begin by looking at aggregate results over the entire Pacific Northwest domain, for the full 2003 calendar year, with the data available from late 2002 used only as training data, to allow us to create forecasts starting in January. The following section will then look at some more specific examples of results for individual locations and/or times.

#### **3.1 Results for the Pacific Northwest**

In assessing probabilistic forecasts of wind speed, we aim to maximize the sharpness of the predictive PDFs subject to calibration (Gneiting, Balabdaoui, and Raftery 2007). Calibration refers to the statistical consistency between the forecast PDFs and the observations. To assess calibration, we consider Figure 2, which shows the verification rank histogram for the raw ensemble forecast and probability integral transform (PIT) histograms for the BMA forecast distributions. In both cases, a more uniform histogram indicates better calibration. The verification rank histogram plots the rank of each observed wind speed relative to the eight ensemble member forecasts. If the observation and the ensemble members come from the same distribution, then the observed and forecasted values are exchangeable so that all possible ranks are equally likely. The PIT is the value that the predictive cumulative distribution function attains at the observation and is a continuous analog of the verification rank.

For our data, the verification rank histogram illustrates the lack of calibration in the raw ensemble, which is underdispersed, similarly to results reported by Eckel and Walters (1998), Hamill and Colucci (1998), and Mullen and Buizza (2001) for other ensembles. From the PIT histograms for the BMA forecast distributions, all five methods of parameter estimation gave similar results, in all cases substantially better calibrated than the raw ensemble.

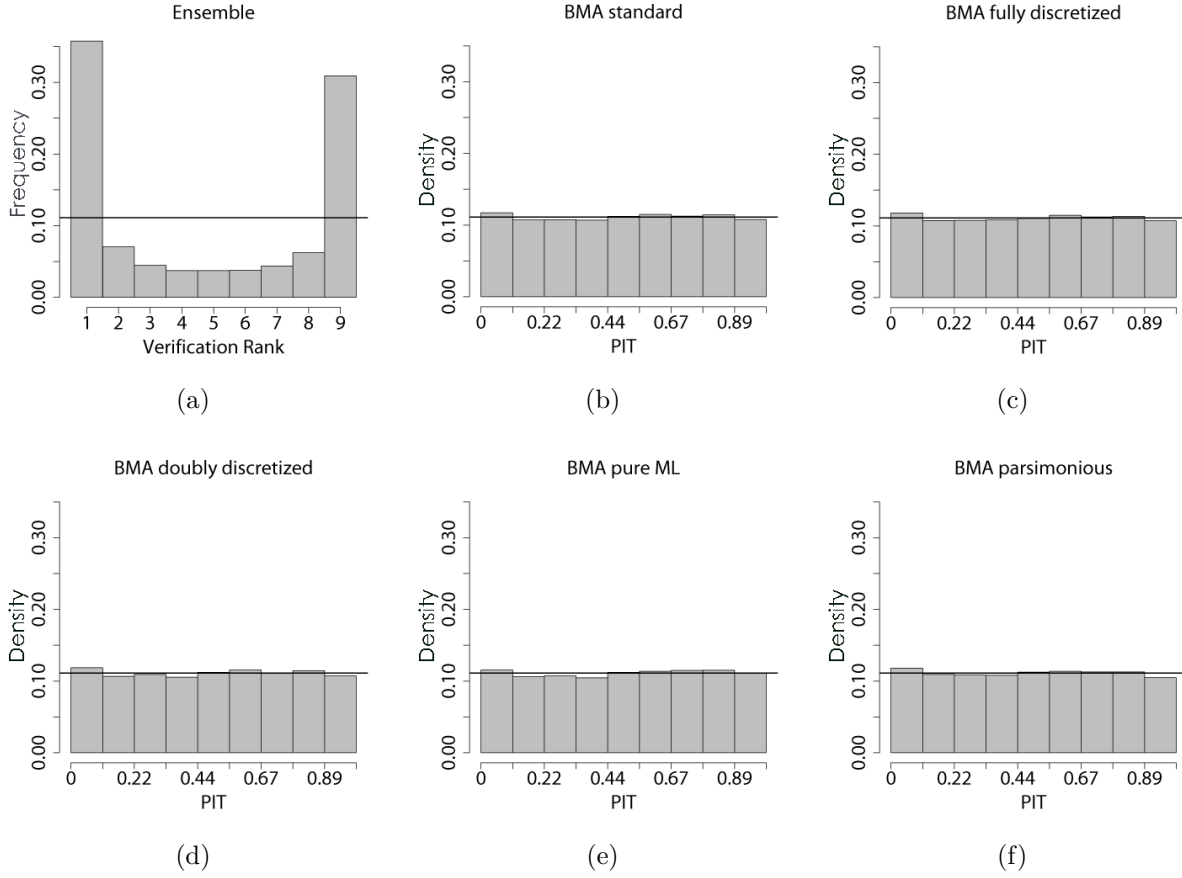


Figure 2: Calibration checks for probabilistic forecasts of wind speed over the Pacific Northwest in 2003. (a) Verification rank histogram for the raw ensemble, and PIT histograms for the BMA forecast distributions estimated using (b) the standard method, (c) the fully discretized method, (d) the doubly discretized method, (e) the pure maximum likelihood method, and (f) the parsimonious method.

If the eight-member raw ensemble were properly calibrated, there would be a  $\frac{1}{9}$  probability of the wind speed observation falling below the ensemble range, and a  $\frac{1}{9}$  probability of it falling above the ensemble range. As such, to allow direct comparisons to the raw ensemble, we will consider  $\frac{7}{9}$  or 77.8% central prediction intervals from the BMA PDF. Table 1 shows the empirical coverage of 77.8% prediction intervals, and the results echo what we see in the verification rank and PIT histograms. The raw ensemble was highly uncalibrated. The BMA intervals were well calibrated. The table also shows the average width of the prediction intervals, which characterizes the sharpness of the forecast distributions. While the raw ensemble provides a narrower interval, this comes at the cost of much poorer calibration.

Scoring rules provide summary measures of predictive performance that address calibration and sharpness simultaneously. A particularly attractive scoring rule for probabilistic forecasts of a scalar variable is the continuous ranked probability score (CRPS), which gen-

Table 1: Mean continuous ranked probability score (CRPS) and mean absolute error (MAE), and coverage and average width of 77.8% central prediction intervals for probabilistic forecasts of wind speed over the Pacific Northwest in 2003. Coverage in percent, all other values in knots. The MAE refers to the point forecast given by the median of the respective forecast distribution.

Forecast	CRPS	MAE	Coverage	Width
Climatology	2.972	4.143	77.8	13.00
Ensemble	2.918	3.544	33.3	3.98
BMA standard	2.397	3.382	77.5	10.32
BMA fully discretized	2.397	3.382	77.3	10.27
BMA doubly discretized	2.397	3.383	77.3	10.28
BMA pure MLE	2.392	3.376	77.3	10.20
BMA parsimonious	2.398	3.384	77.7	10.36

eralizes the mean absolute error (MAE), and can be directly compared to the latter. It is a proper scoring rule and is defined as

$$\text{crps}(P, x) = \int_{-\infty}^{\infty} (P(y) - I\{y \geq x\})^2 dy = E_P|X - x| - \frac{1}{2} E_P|X - X'|, \quad (17)$$

where  $P$  is the predictive distribution, here taking the form of a cumulative distribution function,  $x$  is the observed wind speed, and  $X$  and  $X'$  are independent random variables with distribution  $P$  (Grimmett et al. 2006; Wilks 2006; Gneiting and Raftery 2007). Both CRPS and MAE are negatively oriented, that is, the smaller the better.

Table 1 shows CRPS and MAE values for climatology, that is, the marginal distribution of observed wind speed across space and time for the dataset, the raw ensemble forecast, and the BMA forecasts, all in units of knots. A point forecast can be created from the BMA forecast distribution by finding the median of the predictive distribution, and the MAE refers to this forecast. Similarly, we show the MAE for the median of the eight-member forecast ensemble, with the results for BMA being by far the best. The results for the CRPS were similar, in that BMA outperformed the raw ensemble and climatology.

These results show that all the parameter estimation methods considered performed similarly. While the pure maximum likelihood method gave very slightly better results, it was much slower computationally. We therefore recommend the use of the parsimonious method, which is the simplest in terms both of number of parameters to be estimated and of computational needs.

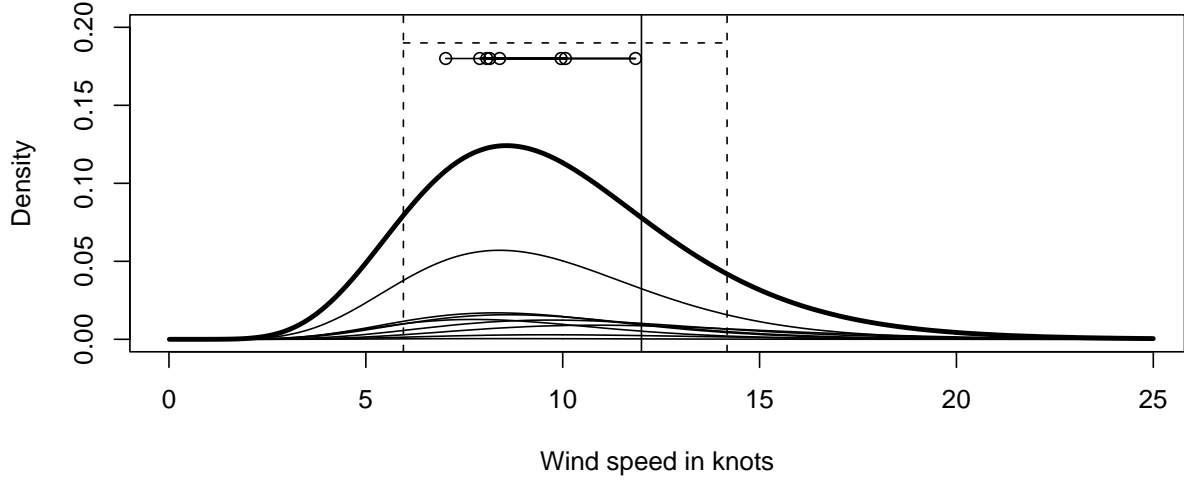


Figure 3: 48-hour ahead BMA predictive PDF for maximum wind speed at Shelton, Washington on 7 August 2003. The upper solid curve is the BMA PDF. The lower curves are the components of the BMA PDF, namely, the weighted contributions from the ensemble members. The dashed vertical lines represent the 11th and 89th percentiles of the BMA PDF; the dashed horizontal line is the respective prediction interval; the circles represent the ensemble member forecasts; and the solid vertical line represents the verifying observation.

### 3.2 Examples

To illustrate the BMA forecast distributions for wind speed, we show an example, on 7 August 2003 at Shelton, Washington. Figure 3 shows the ensemble values, the BMA component distributions, the BMA PDF, the BMA central 77.8% forecast interval, and the observation. The observed wind speed of 12 knots fell just above the ensemble range, while it was within the range of the BMA interval.

Figure 4 shows maps of the BMA median and BMA 90th percentile upper bound forecast for 7 August 2003. If we compare to the ensemble forecast in Figure 1 we see that the general spatial structure is largely preserved in the BMA forecast. Figure 5 shows the verifying wind observations over the Pacific Northwest on 7 August 2003. It is evident that the raw ensemble was underforecasting in a number of areas where the BMA forecast was not.

Finally, we compare the BMA forecasts at Shelton, Washington over the 2003 calendar year to the raw ensemble forecast and the station climatology, that is, the marginal distribution of all observed values at Shelton over the time period considered. Table 2 shows CRPS and MAE scores along with prediction interval coverage and average width for station climatology, the raw ensemble, and BMA at this location. The BMA forecast showed

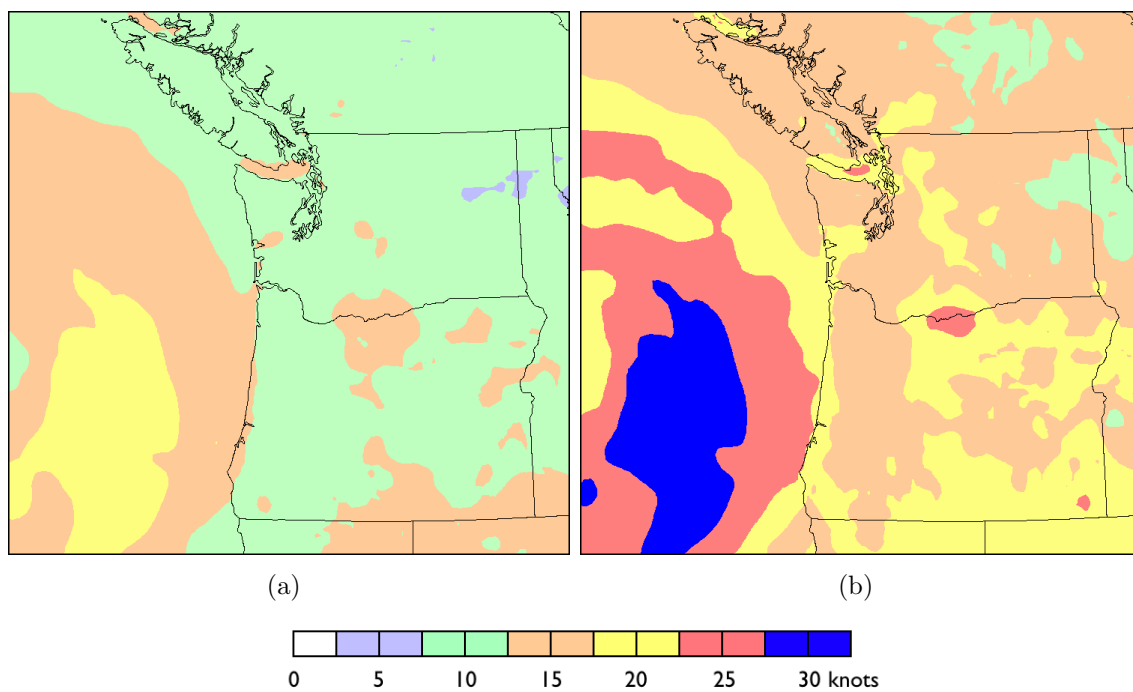


Figure 4: 48-hour ahead (a) BMA median forecast, and (b) BMA 90th percentile forecast of maximum wind speed over the Pacific Northwest on 7 August 2003.

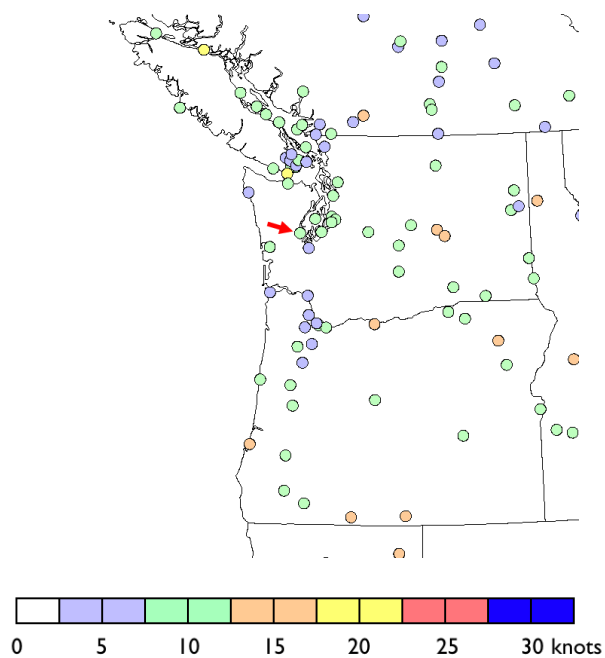


Figure 5: Observed maximum wind speeds at meteorological stations over the Pacific Northwest on 7 August 2003. The arrow indicates the station at Shelton, Washington.

Table 2: Mean continuous ranked probability score (CRPS) and mean absolute error (MAE), and coverage and average width of 77.8% central prediction intervals for probabilistic forecasts of wind speed at Shelton, Washington in 2003. Coverage in percent, all other values in knots. The MAE refers to the point forecast given by the median of the respective forecast distribution.

Forecast	CRPS	MAE	Coverage	Width
Station climatology	2.64	3.82	77.8	12.0
Ensemble	2.80	3.49	35.1	4.2
BMA	2.14	3.01	78.9	10.0

substantially better CRPS and MAE than both station climatology and the raw ensemble. Furthermore, BMA gave sharper intervals on average than station climatology, and a better calibrated interval than the raw ensemble.

It should be noted that wind forecasting is notoriously difficult. While in this example BMA outperformed station climatology, that is not to be taken for granted, and may not be the case at stations at which there is substantial topography at subgrid scales (Gneiting et al. 2008).

## 4 Discussion

We have shown how to apply BMA to wind speed forecasts. This provides a statistical postprocessing method for ensembles of numerical weather predictions that yields a full predictive distribution for maximum wind speed. In our experiments with the University of Washington mesoscale ensemble, the BMA forecast PDFs were better calibrated than the raw ensemble, which was underdispersive. The BMA median forecast had lower MAE than the ensemble median, and the BMA forecast PDFs had substantially lower CRPS than the raw ensemble or climatology.

There are a number of potential wind quantities of interest. Our method was developed for forecasting maximum wind speed over a particular time interval. Instantaneous wind speed, or maximum wind speed over some other time interval, could well have different distributional properties. These differences could possibly be accounted for by fitting a gamma distribution to some power transformation of the wind speed, rather than to the raw wind speed. The power transformation appropriate for any given situation would need to be determined empirically.

Nielsen et al. (2004) presented a probabilistic forecasting method based on correcting quantiles of an ensemble, while other approaches have used climatology conditioned on ensemble forecasts (Roulston et al. 2003) or on numerical weather prediction model output (Bremnes 2006). The model based approach of BMA allows for the fitting of a predictive distribution using sparse training data. By modeling the relation between forecasts and observations, BMA allows for the creation of forecast distributions in situations where the raw forecasts may not have any close analogues in the training set.

Our results showed no appreciable difference in performance between parameter estimation methods that took into account the discretization of the data and those that did not. A detailed simulation study looking at the effects of discretization for meteorological data was carried out by Cooley et al. (2007). Perrin et al. (2006) found that discretized wind speed values can result in erroneously low standard errors for parameter estimates. However, we do not look at standard errors of our parameter estimates here, as we are interested primarily in prediction, and not in parameter estimation for its own sake.

While our implementation has been for a situation where the ensemble members come from clearly distinguishable sources, it is easily modifiable to deal with situations in which ensemble members come from the same model, differing only in some random perturbations, such as the global ensembles that are currently used by the National Centers for Environmental Prediction and the European Centre for Medium-Range Weather Forecasts (Buizza et al. 2005). In these cases, members coming from the same source should be treated as exchangeable, and thus should have equal weight and equal BMA parameter values across members. As our recommended parsimonious method already constrains the mean and standard deviation parameters to be equal across all members, the only change that would need to be made would be to add the constraint that the BMA weights be equal, as described by Raftery et al. (2005, p. 1170) and Wilson et al. (2007).

Our approach has not incorporated temporal autocorrelation. While wind speeds in successive time periods are certainly autocorrelated, our method implicitly addresses forecast errors rather than raw wind speeds, because it models the predictive distribution conditional on the numerical forecasts. Exploratory work has shown that there is little or no autocorrelation in forecast errors of wind speeds, so the numerical forecasts effectively take care of the autocorrelation in the wind speeds themselves. To incorporate temporal autocorrelation, the method would have to be made considerably more complicated, and it seems unlikely that this would improve its predictive performance appreciably.

Our method produces wind speed forecasts at individual locations, which is the focus of many, and possibly most applications. As a result we have not had to model spatial correlation between wind speeds, although these definitely are present. In applications that

involve forecasting wind speeds at more than one location simultaneously, it would be vital to take account of spatial correlation. These include forecasting maximum wind speeds over an area or trajectory, for example for shipping or boating, and forecasting the total energy from several wind farms in a region. Methods for probabilistic weather forecasting at multiple locations simultaneously have been developed for temperature (Gel, Raftery, and Gneiting 2004; Berrocal, Raftery, and Gneiting 2007), for precipitation (Berrocal, Raftery, and Gneiting 2009), and for temperature and precipitation simultaneously (Berrocal et al. 2007). These methods could possibly be extended to wind speeds.

Our method estimates a single set of parameters across the entire domain. Nott et al. (2001) noted that localized statistical postprocessing can address issues of locally varying biases in numerical weather forecasts. A localized version of BMA for temperature, based on taking sets of forecasts and observations within a carefully selected neighborhood, has shown substantial improvements over the global version (Mass et al. 2008), and it is likely that similar improvements would be seen for a localized version of BMA for wind speed.

Either the global or localized parameter estimation can be used to create probabilistic forecasts of wind speed on a spatial grid. These methods have already been implemented for temperature and precipitation and provide real-time probabilistic weather forecasts over the Pacific Northwest (Mass et al. 2008), which are available to the general public at <http://probcast.washington.edu>. We intend to make similar, gridded probabilistic forecasts for wind speed available also. Additionally, for specialized uses where only a single location is of interest, such as a wind farm or a windsurfing or sailing location, BMA parameters could be fit using the same methodology as described in this paper, but restricting it to use only data from that particular location.

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